CMC: Calculation of Small Color Differences for Acceptability

Developed by AATCC Committee RA36 in 1989; reaffirmed 1990; editorially revised and reaffirmed 1991, 2005; revised and reaffirmed 1992; revised 1998. Partly related to ISO 105-J03.

1. Purpose and Scope

1.1 The CMC (ℓ:c) formula is a modification of the corresponding CIELAB color-difference formula. It has color-difference symbol ΔE_{cmc}. Even though use of the CIE 1976 L*a*b* (CIELAB) equation provides industry with a 'standard' means of calculating and communicating color differences of trials from their standards (see 8.1), it is recognized that CIELAB color-difference values (ΔE^*) correlate poorly with visual assessments. The lack of correlation is caused by the non-uniformity of CIELAB color space (see Fig. 1). Significantly improved correlation between visual assessments and instrumentally measured color differences is given by the CMC (ℓ:c) color-difference formula (see 7.1 to 7.3). The improvement in correlation between visual and instrumental color-difference reports yielded by the CMC (ℓ:c) equation will usually permit the adoption of a single-number tolerance for judging the acceptability of a color match in most situations, regardless of both the color of the standard and the direction of the color difference of any trial from it. Another equation, CIE94 has been considered, but it has not shown any significant improvement over CMC (see 8.2).

1.2 The CMC (ℓ:c) formula retains, in modified form, the partitioning of overall color difference into differences in lightness, chroma, and hue components (see Fig. 2) which is present in the CIELAB color-difference formula. Using the ellipsoidal semi-axes (ℓS_L, cS_C, and S_H) makes the CMC (ℓ:c) equation suitable for a wide range of uses.

2. Principle

2.1 The CMC (ℓ:c) modification to CIELAB provides a unit of measurement for the acceptance volume about a standard color. This volume takes the shape of an ellipsoid whose semi-axes are ℓS_L, cS_C, and S_H in the directions of lightness, chroma, and hue differences, respectively, in CIELAB color space. The CMC (ℓ:c) formula varies the ratio of the lengths of these three semi–axes systematically throughout CIELAB color space according equality of lengths with visually equal color differences, regardless of both the color of the standard and direction of difference of any trial from it. Around any given standard color, the ratio S_L:S_C:S_H is fixed regardless of the end use of the trials.

3. Terminology

3.1 **CIE 1976 L*a*b* equation,** n.—a commonly used equation which transforms CIE tristimulus values into a three-dimensional opposed color space. Generally abbreviated as CIELAB (see 8.1).

3.2 **CMC unit,** n.—*in color difference evaluation*, a measure of acceptability expressed in terms of the boundary for the CMC acceptability ellipsoid of ΔE_{cmc} = 1.0.

3.2.1 CMC is an acronym for the Colour Measurement Committee of The Society of Dyers and Colourists which was largely responsible for the development of the CMC (ℓ:c) equation.

3.3 **commercial factor (cf),** n.—*in color difference evaluation*, a tolerance (specified in terms of ΔE_{cmc} units) which adjusts all axes of the unit CMC volume equally to create a volume of acceptance for commercial use.

3.4 $\boldsymbol{\Delta E_{cmc}}$**,** n.—*in color difference evaluation*, a single number defining the total color difference in CMC units of a trial from a standard.

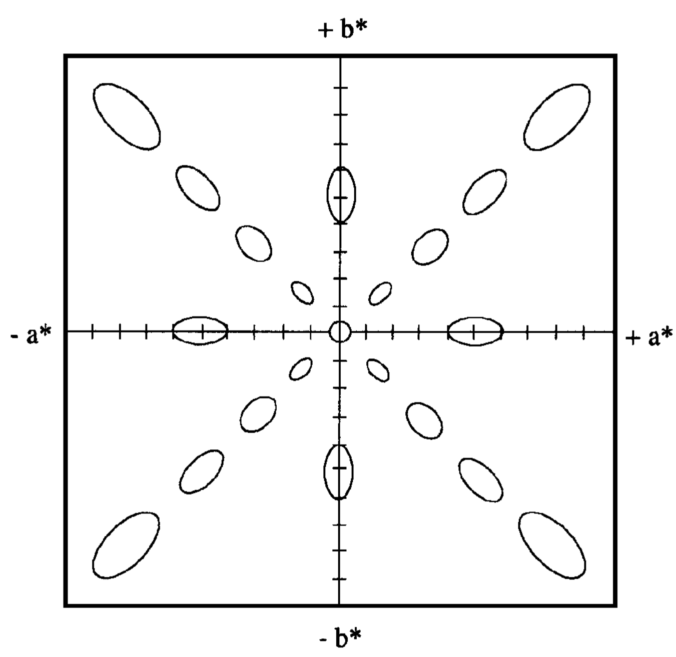

Fig. 1—CMC(ℓ:1) Unit ellipsoid cross sections in CIELAB a*b* diagram.

If CIELAB were uniform, these sections would be equal-sized circles.

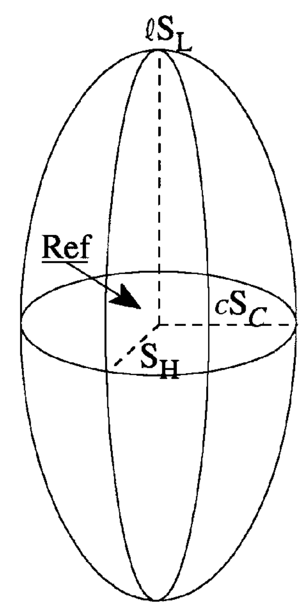

Fig. 2—Unit volume of acceptance.

Distance from standard to surface is one CMC(ℓ:c) unit.

3.5 **semi-axes (ℓS_L, cS_C, and S_H)**, n.—*in color difference evaluation*, individual dimensions of the CMC volume which are used to calculate a ΔE_{cmc} value.

3.6 **volume of acceptability,** n.—*in color difference evaluation*, the volume of the ellipsoid obtained when each semi-axis (ℓS_L, cS_C, and S_H) is multiplied by cf—creating an agreed upon volume describing the limits of commercial acceptability for the color difference about a standard.

4. Procedure of Calculation

4.1 Calculation of CIELAB values.

4.1.1 Calculate the CIELAB L^*,C_{ab}^*, and h_{ab} values from the CIE tristimulus values X, Y, Z of each specimen using equations 1, 2, 3 and 4:

$$L^* = 116\,(Y/Y_n)^{1/3} - 16 \quad (1)$$

if

$Y/Y_n > 0.008856$

but

$$L^* = 903.3\,(Y/Y_n) \quad (2)$$

if

$Y/Y_n \leq 0.008856$

$a^* = 500[f(X/X_n) - f(Y/Y_n)]$

$b^* = 200[f(Y/Y_n) - f(Z/Z_n)]$

where

$f(X/X_n) = (X/X_n)^{1/3}$

if

$X/X_n > 0.008856$

but

$f(X/X_n) = 7.787(X/X_n) + 16/116$

if

$X/X_n \leq 0.00856$

$f(Y/Y_n) = (Y/Y_n)^{1/3}$

if

$Y/Y_n > 0.008856$

but

$f(Y/Y_n) = 7.787(Y/Y_n) + 16/116$

if

$Y/Y_n \leq 0.008856$

$f(Z/Z_n) = (Z/Z_n)^{1/3}$

if

$Z/Z_n > 0.008856$

but

$f(Z/Z_n) = 7.787(Z/Z_n) + 16/116$

if

$Z/Z_n \leq 0.008856$

$$C^*_{ab} = (a^{*2} + b^{*2})^{1/2} \quad (3)$$

$$h_{ab} = \arctan b^*/a^* \quad (4)$$

expressed on a 0-360° scale with the a^* + axis being 0° and the b^* axis at 90°.

4.1.2 For these equations, X_n, Y_n, and Z_n are the CIE tristimulus values for the chosen illuminant and observer. The preferred combination is CIE standard illuminant D_{65} and the CIE 1964 supplementary standard (10°) observer. Values for this combination and for some others are given in Table I. Values for combinations not in Table I should be obtained by referring to ASTM E 308 (see 8.3) or, only if the required combination is not therein, CIE 15.2 (see 8.1).

Table I—Tristimulus Values for Four Illuminant-Observer Combinations

Illuminant-Observer Combinations	Tristimulus Value		
	X_n	Y_n	Z_n
D_{65}/10°	94.811	100.000	107.304
D_{65}/2°	95.047	100.000	108.883
C/10°	97.285	100.000	116.145
C/2°	98.074	100.000	118.232

4.2 Calculation of CIELAB color difference values.

4.2.1 Calculate the CIELAB ΔL^*, ΔC^*_{ab}, and ΔH^*_{ab} color difference values as follows, where subscripts S and R refer to the specimen and reference CIELAB values:

$\Delta L^* = L^*_S - L^*_R$

$\Delta a^* = a^*_S - a^*_R$

$\Delta b^* = b^*_S - b^*_R$

$\Delta C^*_{ab} = C^*_{ab,S} - C^*_{ab,R}$

$\Delta E^*_{ab} = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2}$

$\Delta H^*_{ab} = pq[\Delta E^*_{ab})^2 - (\Delta L^*)^2 - (\Delta C^*_{ab})^2]^{1/2}$

where $p = 1$ if $m \geq 0$ or $p = -1$ if $m < 0$ and $q = 1$ if $|m| \leq 180$ or $q = -1$ if $|m| > 180$

where $m = h_{ab,S} - h_{ab,R}$ in which |…| indicates that the positive value is to be used regardless of the sign of the expression between the two lines

4.2.2 An equivalent method of calculating ΔH^*_{ab} is to use Eq. 5:

$$\Delta H^*_{ab} = t[2(C^*_{ab,S}\,C^*_{ab,R} - a^*_S\,a^*_R - b^*_S\,b^*_R]^{1/2} \quad (5)$$

where $t = 1$ if $a^*_S\,b^*_R \leq a^*_R\,b^*_S$ or $t = -1$ if $a^*_S\,b^*_R > a^*_R\,b^*_S$

4.3 Calculation of ΔE_{cmc}.

4.3.1 Calculate the color difference in CMC (ℓ:c) units using Eq. 6:

$$\Delta E_{cmc} = \left[\left(\frac{\Delta L^*}{\ell S_L}\right)^2 + \left(\frac{\Delta C^*_{ab}}{cS_C}\right)^2 + \left(\frac{\Delta H^*_{ab}}{S_H}\right)^2\right]^{1/2} \quad (6)$$

where for the standard:

$$\text{for } L^*_S > 16 \quad S_L = \frac{0.040975L^*}{1 + 0.01765L^*}$$

$$\text{for } L^*_S \leq 16 \quad S_L = 0.511$$

$$S_C = \frac{0.0638C^*_{ab}}{1 + 0.0131C^*_{ab}} + 0.638$$

$$S_H = (FT + 1 - F)S_C$$

where

$$F = \left[\frac{(C^*_{ab})^4}{(C^*_{ab})^4 + 1900}\right]^{1/2}$$

and

$$T = 0.36 + abs\,[0.4\cos(35 + h_{ab})]$$

unless h_{ab} is between 164° and 345° then

$$T = 0.56 + abs\,[0.2\cos(168 + h_{ab})]$$

For the last two equations, '*abs*' indicates the absolute—i.e., positive value—of the term inside the square brackets.

Note: If required, the components of CMC (ℓ:c) color difference (ΔL_{cmc}, ΔC_{cmc}, and ΔH_{cmc}) may be calculated using the terms in the round brackets in the first line of the above equation, namely:

$$\Delta L_{cmc} = \frac{\Delta L^*}{\ell S_L}$$

$$\Delta C_{cmc} = \frac{\Delta C^*_{ab}}{cS_C}$$

$$\Delta H_{cmc} = \frac{\Delta H^*_{ab}}{S_H}$$

4.3.1.1 When ℓ = 2.0 this equation fixes the ratio of the three terms (S_L:S_C:S_H:) to correlate with visual assessment of typical textile samples. Other values of ℓ may be required in cases where the surface characteristics are significantly different. For example, other values may be required when measuring very dark trials, but the user should assume an ℓ value of 2.0 until actual results indicate a need for adjustment. In practice, "c" is always set to unity and it may be omitted from the equation.

4.3.2 Concept of unit volume/tolerance. The equation for $\Delta E_{cmc} \leq 1.0$ describes an ellipsoidal volume, with axes in the direction of lightness, chroma and hue centered about a standard. Ellipsoid semi-axis lengths of ℓS_L, cS_C and S_H calculated for a given standard describe a unit volume of acceptance within which all samples are less than 1.0 ΔE_{cmc} unit from the standard.

NOTE: Size and orientation of the diagram shown in Fig. 2 varies considerably depending on location of standard in color space and is used here as a means of conceptualizing the semi-axes.

4.3.2.1 This volume—and the size and ratio of its semi-axes—become the basis for the establishment of an appropriately sized volume of acceptability for a given commercial situation by the application of a commercial factor (cf) to all dimensions.

4.3.2.2 The Volume of Acceptance is defined by $\Delta E_{cmc} \leq$ cf.

4.4 Reporting of results. The use of illuminant D_{65} and 10° observer calculations and (ℓ:c) ratio of 2:1 are recommended as the standard for calculating ΔE_{cmc} values. If other illuminants, observers or CMC (ℓ:c) ratios are used they must be specified as part of the value (i.e., $\Delta E_{cmc(1.37:1),\ C/2°}$ = 1.56 would denote using a 1.37:1 ℓ:c ratio, with CIELAB values calculated for illuminant C/2° observer).

5. Interpretation of Results

5.1 ΔE_{cmc} is a single number which represents the number of CMC color difference units of a trial from a standard. Any trial which is compared to its standard will fall into one of three categories:

5.1.1 If ΔE_{cmc} is less than the agreed commercial factor, the trial is acceptable;

5.1.2 If ΔE_{cmc} is close to the agreed commercial factor, the trial is borderline;

5.1.3 If ΔE_{cmc} is greater than the agreed commercial factor, the trial is unacceptable.

5.2 The semi-axes ℓS_L, cS_C and S_H provide a means of describing the "acceptability" of each dimension of color difference (lightness, chroma, hue) in terms of CMC units in each dimension.

5.2.1 In applications which require rating within or about a critical tolerance, multiple levels consisting of ellipsoidal volumes representing various levels of color difference may be generated. This will result in a set of concentric volumes/tolerances which provide a uniform grading system when correlated to and are associated with a predefined set of terms.

6. Precision and Bias

6.1 An inter-laboratory trial to determine precision and bias of this method was begun in 1998. Results should be available by late 1998.

7. References

7.1 BS6923:1988, British Standard Method for Calculation of Small Colour Difference. Available from British Standards Institution, 2 Park Street, London W1A 2BS, England.

7.2 Clarke, F. J. J., R. McDonald and B. Rigg, Modification to the JPC79 Colour-Difference Formula, *Journal of The Society of Dyers and Colourists*, Vol. 100, 1984, p128-132 and p281-282.

7.3 McDonald, R., Acceptability and Perceptibility Decisions Using the CMC Colour Difference Formula, *Textile Chemist and Colorist*, Vol. 20, No. 6, 1988, p31-37, and Errata, Vol. 20, No. 8, 1988, p10.

8. Notes

8.1 CIE Publication No. 15.2, *Colorimetry*, Second Edition, 1986. Available from U.S. National Committee, CIE c/o Mr. Robert A. McCully, Philips Lighting Co., P.O. Box 6800, Somerset NJ 08875-6800.

8.2 McDonald, Roderick and Kenneth J. Smith, "CIE94—A New Colour Difference Formula," *Journal of the Society of Dyers and Colourists*, Vol. 111, No. 12, Dec. 1995, p376.

8.3 ASTM E 308, *Practice for Computing the Colors of Objects by Using the CIE System*, available from ASTM, 100 Barr Harbor, West Conshohocken PA 19428.

8.4 Simon, F.T., *die Farbe*, Vol. 10, 1961, p225.

8.5 Harold, R. W., *Textile Chemist and Colorist*, Vol. 19, No. 12, 1987, p23.

8.6 The value of 1.0 for the constant c in the formula is mandatory in AATCC Method 173. It is also recommended in the corresponding ISO test method 105-J03 but is given in a note and thereby not mandatory. AATCC has petitioned ISO to make it also mandatory. If and when this happens the two methods can again be considered technically equivalent.

Appendix A. Computer Program

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10      'CMC (L:C) COLOR DIFFERENCE FORMULA
20      '##############################################
30      'Input data and LPRINT results
40      '##############################################
50      INPUT "Input CMC (l:c) weighting factors 'l', 'c' "; L, C
60      INPUT "Input X,Y,Z of standard"; X(1), X(2), X(3)
65      LPRINT "X,Y,Z of standard"; X(1), X(2), X(3): GOSUB 160: L1=CL: A1=CA: B1=CB
70      INPUT "Input X,Y,Z of trial"; X(1), X(2), X(3)
75      LPRINT "X,Y,Z of trial "; X(1), X(2), X(3): GOSUB 160: L2=CL: A2=CA: B2=CB
80      GOSUB 230
90      LPRINT "L*,a*,b*, Hue angle of standard "; L1, A1, B1, H1
100     LPRINT "L*,a*,b*, Hue angle of trial "; L2, A2, B2, H2
110     LPRINT "DL/lSl DC/cSc DH/Sh DEcmc("; l; ":" c ")"
120     LPRINT DL; DC; DH; DE: LPRINT : GOTO 60
130     '##############################################
140     'Calculate L*, a*, b* values (d65/10)
150     '##############################################
160     X(1) = X(1) / 94.811: X(2) = X(2) / 100: X(3) = X(3) / 107.304
170     FOR I = 1 TO 3: IF X(I)< 8.856001E-03 THEN FX(I) =7.787*X(I)+16 /116
        ELSE FX(I)=X(I) ^ (1 / 3)
180     NEXT
190     CL = 116 * FX(2) - 16: CA = 500 * (FX(1) - FX(2)): CB = 200 * (FX(2) - FX(3)): RETURN
200     '##############################################
210     'Calculate CMC color difference
220     '##############################################
230     DL=L2-L1: C1=SQR(B1*B1+A1*A1): C2=SQR(B2*B2+A2*A2): DC=C2-C1
240     Sl = DL*DL+(A2-A1)*(A2-A1)+(B2-B1)*(B2-B1)
250     DH =0: AA=Sl-DL*DL-DC*DC: IF AA <= 0 THEN 260 ELSE DH = SQR(AA)
260     IF (A2*B2) =0 THEN 280 ELSE H2 =180-SGN(B2)*90 -ATN(A2/B2)*57.3
270     GOTO 300
280     BB2 = SGN(ABS(B2)): AA2 = SGN(A2 + B2)
290     H2 = 90 * (BB2 - AA2 + 1)
300     IF (A1*B1) =0 THEN 320 ELSE H1 =180-SGN(B1)*90-ATN(A1/B1)*57.3
310     GOTO 340
320     BB1 = SGN(ABS(B1)): AA1 = SGN(A1 + B1)
330     H1 = 90 * (BB1 - AA1 + 1)
340     IF H1 <= 164 OR H1 >= 345 THEN 350 ELSE GOTO 360
350     T = .36 + ABS(.4 * COS((H1 + 35) / 57.3)): GOTO 370
360     T = .56 + ABS(.2 * COS((H1 + 168) / 57.3))
370     Sl = .040975 * L1 / (1 + .01765 * L1): IF L1 <= 16 THEN LET Sl = .511
380     Sc =.0638*C1/(1+.0131*C1)+.638: F =SQR(C1^ 4/(C1^4+1900)): Sh =Sc*(T*F+1-F):
        DL =DL/(l*Sl): DC =DC/(c*Sc): DH=DH/Sh
385     DA = H2 - H1: IF ABS(DA) > 180 THEN Y1 = -1 ELSE Y1 = 1
386     Y2 = Y1 * DA: IF Y2 <= 0 THEN DH = -DH
390     DE = SQR(DL * DL + DC * DC + DH * DH)
400     RETURN
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Appendix B. Representative Test Data

To help check computer programs giving ΔE_{cmc} values from the CMC equation, some representative test data are given in Table II. The data are for illuminant D_{65} and the 10° observer using $X_n = 94.811$, $Y_n = 100.00$, $Z_n = 107.304$ (from Table I). The six reference pair colors shown are red, blue, yellow, green, grey, and another red. The ℓ:c ratio used is 2:1.

Table II—Test Data for the CMC(2:1) Equation (D_{65}/10)

Pair	Tristimulus Values			CIELAB Values			ΔE_{cmc}
	X	*Y*	*Z*	*L**	*a**	*b**	
1	69.556	70.797	67.146	87.39	5.32	7.19	0.42
	68.614	69.698	65.942	86.85	5.59	7.29	
2	53.180	57.467	66.036	80.44	−3.35	−3.84	0.45
	54.385	58.760	67.111	81.16	−3.35	−3.52	
3	63.089	67.667	23.126	85.84	−2.45	55.67	0.27
	61.950	66.366	22.565	85.18	−2.26	55.52	
4	23.178	28.245	21.074	60.11	−15.42	14.97	0.97
	21.896	27.060	20.137	59.03	−16.64	14.86	
5	12.938	13.590	16.071	43.64	0.35	−3.39	0.81
	12.168	12.737	15.221	42.36	0.64	−3.68	
6	14.640	11.100	11.060	39.75	27.95	2.35	2.33
	14.520	11.190	12.220	39.90	26.57	−0.57	